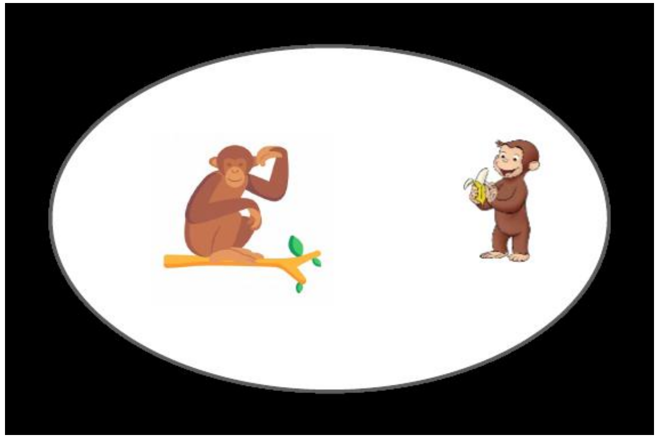
Theories
(of the world)
(HP2T)

Index

- **From analogical to linguistic representations**
- Logical languages
- Words, alphabet
- Assertional languages
- Assertions
- Assertional theories
- Key notions

From analogic to linguistic representations

- There is a tree
- There is a banana
- The monkey is eating a banana
- The monkey is sitting on a tree
- The monkey is scratching its head



Observation 1. Which monkey? There are two monkeys! The language used in the linguistic representation is ambiguous! We need to provide a formalization of the linguistic representation on the left.

Observation 2. Model theory provides us with a formal mechanism for formalizing the content of the analogical representation on the right.

Observation 3. Still the language used is full of ambiguities. We need to formalize linguistic representations in way that univocally reflect the contents of analogical representations, when formalized in model theory. We need both representations!

From models to theories

Intuition (From models to theories): **Models** depict a specific instance of the world, e.g., the world under observation. This instance is called the **intended model**. Representations should

- … take in input the information encoded in the intended model
- … define an appropriate linguistic representation of the intended model

We call any suitably formalized linguistic representation of an intended model, a **theory** (for that intended model).

Observation (From models to theories). Theories must be constructed by providing the means of linguistically represent each and every element of a model.

Linguistic representations vs. theories

Example (Linguistic representations): Examples of linguistic representatins are all the reference models used in Computer Science (see dedicated lecture). **In none of them the intended model is formalized**. That is, **none of them is a theory** (of the world). For instance:

1. **Natural language** text can be used describe all types of percepts. The syntax is informally defined. The intended model is left implicit relying on the commonsense meaning of words. The interpretation of the speaker's intended model relies exclusively on the personal knowledge of the listener.
2. **ER and EER** models describe properties and relations between etypes. The syntax is formally defined. With respect to natural languages, an important simplification derives from a much clearer and simpler syntax.
3. **Relational databases** represent the properties and relations among entities. The syntax is formally defined. The identification of the intended meaning is improved by the unique names assumption, that is: different words have different meanings, multiple occurrences of the same word have the same meaning.

At different levels, all these linguistic representations suffer from the **problem of ambiguity.**

Index

- From analogical to linguistic representations
- **Logical languages**
- Words, alphabet
- Assertional languages
- Assertions
- Assertional theories
- Key notions

Languages

Definition (Language). A **language** L is defined as

$$L = \langle A, FR \rangle = \{s\}$$

where

- A is an **alphabet**, namely a set of atomic terms, not further decomposable
- FR is a set of **formation rules** which allow to compose sentences from terms
- {*s*} is the set of all the sentences which can be generated by taking the transitive closures of FR over the terms of S.

Observation (language). A language is defined by humans, agreed upon by humans, with goal to communicate among humans.

Example (Language). Any natural language, the language of signs, Java, Python, the graphic notation of ER / EER Graphs, or of tables.

Logical Languages

Observation (Language). Our notion of language is as defined by Chomsky's (see lectures of reference formalisms).

Observation (Logical language, assertional language, entailment language). Our focus is on **logical languages**, that is on languages which are formally defined (see later) and that allow to describe and reason about analogic representations and to reason about them. We call the first type of languages, **assertional languages**, and the second, **entailment languages.**

Example (Assertional language). All the reference models, use an assertional language. Natural language allows to say us much more, but can be restricted to allow only for assertions. Some assertional languages are graphical.

Example (Entailment language). Natural language is an entailment language. Any language which allows to state truth in terms of simpler truths, e.g. using connectives such as "and", "or, "implies, "not" is a logical.

Observation (Logical language, assertional language, entailment language). All the logical languages are type 3 languages in the Chomsky hierarchy.

Index

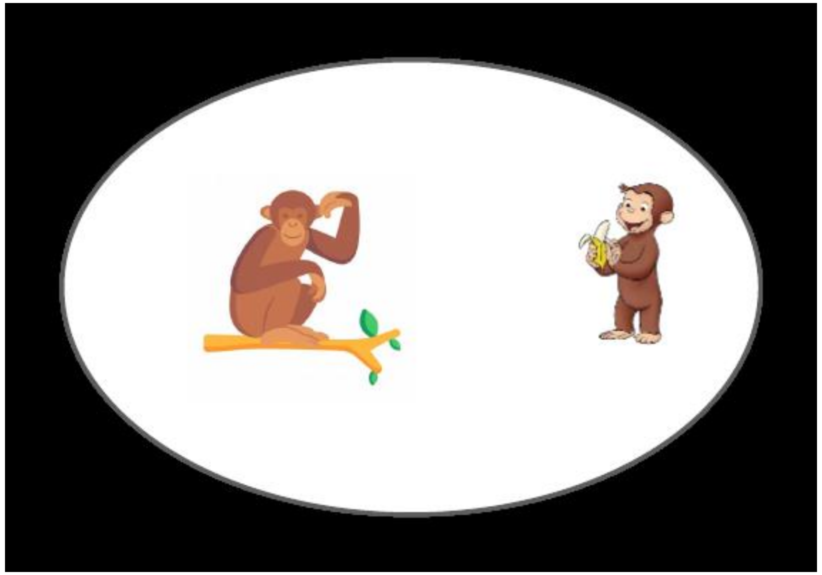
- From analogical to linguistic representations
- Logical languages
- **Words, alphabet**
- Assertional languages
- Assertions
- Assertional theories
- Key notions

Percepts, words, concepts

Intuition (percept, word) A percept, to be a percept, must be linguistically described as such. We say that a percept **p** is **named** or, also, **denoted** by a **word w**, if **p** is the **intended meaning** of **w.** We also say that **p** is the **interpretation** of **w.**

Intuition (concept)[1] A **concept** is the mapping **from a word to the percept it names**. Words are also said to be **lexicalized concepts.**

Example (concept). If the picture on the left is an analogical representation of a domain and intended model, the following are examples of concepts.



- monkey#1: < Chita, "the monkey you see left">
- monkey#2: < The_monkey_on_the_tree,
  "the monkey you see on the tree">
- Monkey: <{monkey#1, monkey#2}>

[1] Miller, G. A., Beckwith, R., Fellbaum, C., Gross, D., & Miller, K. J. (1990). Introduction to WordNet: An on-line lexical database. *Int. journal of lexicography*, *3*(4), 235-244.

Alphabet

Intuition (alphabet). An **alphabet A** is a set of words **w**, that is

A ={w}

Observation (word, alphabet) We need to have words which allow us to name all domain **percepts** of relevance, that is, entities, entity properties, entity relations, etypes, etype properties, etype relations.

Observation (percept, concept). Each percept is associated one and only percept.

Concepts implicit in words (example)

Example (concepts). We have the following:

- a *name* naming an entity (e.g., *Stefania*)
- An *adjective* naming a property of an entity or a set of entities (e.g., *high, beautiful*)
- a *noun* naming a relation between entities or sets of entities (e.g., *friend, owner*)
- a *verb* naming a relation (e.g., to *talk to, to walk with*)
- a *noun* naming a class (e.g., *person, female*)
- a verb naming a class of events (e.g., talking, walking)

Concepts implicit in words (example, continued)

Example (Using words in sentences). We have the following:

- Stefania is beautiful
- Stefania home beautiful (not a sentence!)
- Friends help *(the word friend denotes a class)*
- Stefania is a friend of Mario *(the word friend denotes a relation)*
- Mario has Stefania as a friend *(the word friend denotes a relation, inverse with respect to the previous example)*

Observation 1 (Ambiguity: one word, two different types of percept). For instance, above, the word friend denotes an entity (thing) a class, and a relation

Observation 2 (Ambiguity: one word, two different percepts of the same type). For instance: Java (denotes 2 etype and one entity), car (denotes more than one etype)

Index

- From analogical to linguistic representations
- Logical languages
- Words, alphabet
- **Assertional languages**
- Assertions
- Assertional theories
- Key notions

Assertional Languages

Definition (Assertional Language). Assertional languages are pairs

$$\mathbf{L_{\mathit{a}} = \langle A_{\mathit{a}}, FR \rangle}$$

where

$$\mathbf{A_{\mathit{a}} = \{w\}}$$

is an assertional **alphabet** $\mathbf{A_{\mathit{a}}}$, that is, a set of words **w** that can be used to write assertions, and

$$\mathbf{FR = \{fr\}}$$

is a set of **formation rules fr**, with the restriction that they must be able to describe the facts of domains of interpretation.

From Domains to Assertional Languages

Observation (Domain) As from before, a Domain D is defined as follows:

D= {p}

where percepts p are of three types, that is

D = < U, {C}, {R} >

where: U is a set of **entities**, {C} is a set of **classes** of entities, {R} is a set of n-ary **relations** among entities.

Observation (Assertional Languages). Similarly to domains, an assertional language L_A is defined as follows:

$L_a = <A_a, FR>$

where different types of words describe different types of percepts and the formation rules provuse the means to compose words as a function of how percepts compose to build facts **(compositionality of semantics).**

From Domains to Assertional Languages (continued)

Observation 1 (Choosing percepts) When describing the intended model the key element is the choice of percepts:

- For any percept p ∈ D= {p}, which is **of relevance** to the current modeling task, define a distinct word w ∈ A_a = {w}.

The choice of the percepts defines the expressiveness of world models. Irrelevant classes of percepts can be eliminated, for instance: entities as in ER/ EER models, classes as in relational data bases. In other cases specific classes or relations are dropped.

All percepts can be dropped using words which are assertions naming facts, for instance: from «Near(Paolo,Rocky)» to «NearPaoloRocky» or «A». In this case the set of formation rules FR is empty.

From Domains to Assertional Languages (continued)

Observation (From Percepts to Words) The mapping from percepts to the specific words is in control of the modeler. The usual choice is to choose words whose intuitive meaning are the intended percepts.

Observation 2 (Choosing the formation rules). There are world models which consider the same percepts and are different only in the formation rules they consider. The choice of the formation rules is a modeling decision, driven by the goal of suitably modeling the intended analogical representation. See examples below.

Example – Natural language alphabet and formation rules

Example (Natural language). The origins of Trento on the river-route to Bolzano and the low Alpine passes of Brenner and the Reschen Pass over the Alps **are disputed**. Some scholars maintain it was a Rhaetian settlement: the Adige area was however influenced by neighbouring populations, **including** the (Adriatic) Veneti, the Etruscans and the Gauls (a Celtic population). According to other theories, the **latter** instead founded the city during the 4th century BC.

Observation 1 (Ambiguity of NL). Natural language assertions have informal syntax. They are highly ambiguous because of the complete freedom in using words and constructing setences. Everybody (thinks (s)he) understands them, modulo ambiguity. Used in the requirements phase in most SE projects.

Observation (types of percepts). All types of percepts are allowed.

Example – Pseudo-natural language alphabet and formation rules

Example 1 (Pseudo-natural language). A sequence of statements:

- There is a tree.
- There is a banana.
- The monkey is eating a banana.
- Monkeys eat bananas
- Monkeys are animals

Observation 1 (Pseudo-NL ambiguity). Pseudo natural language assertions have informal an syntax, somehow structured. Thanks to the extremely simplified syntax, everybody understands them, modulo the ambiguity of the words. This type of notation is widely used in high-value applications where the user needs to understand the details of the applications, without having a backgournd in formal logic.

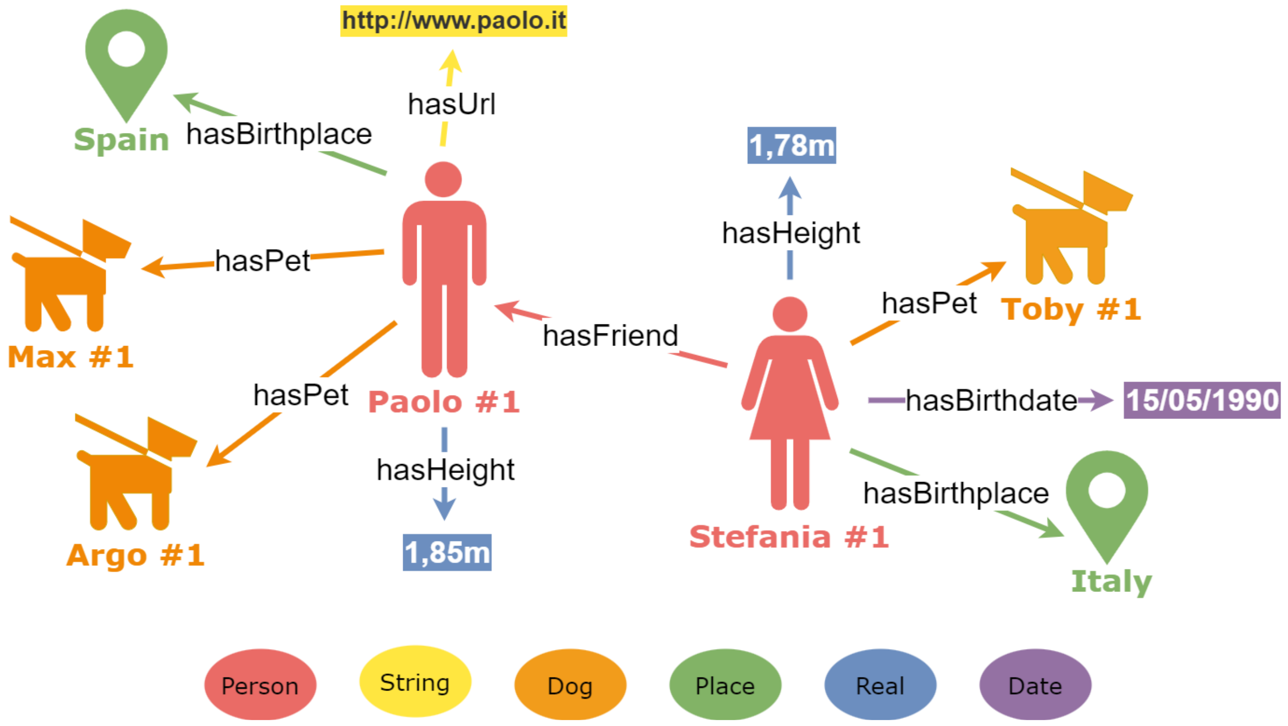
Observation (types of percepts). All types of percepts are allowed.

Example – Database alphabet and formation rules

Employee			
Name	**Role**	**Nationality**	**Supervises**
Fausto	Professor	Italian	Rui
Rui	Student	Chinese	Bisu
Bisu	Student	Indian	-
entity	etype	property	Relation

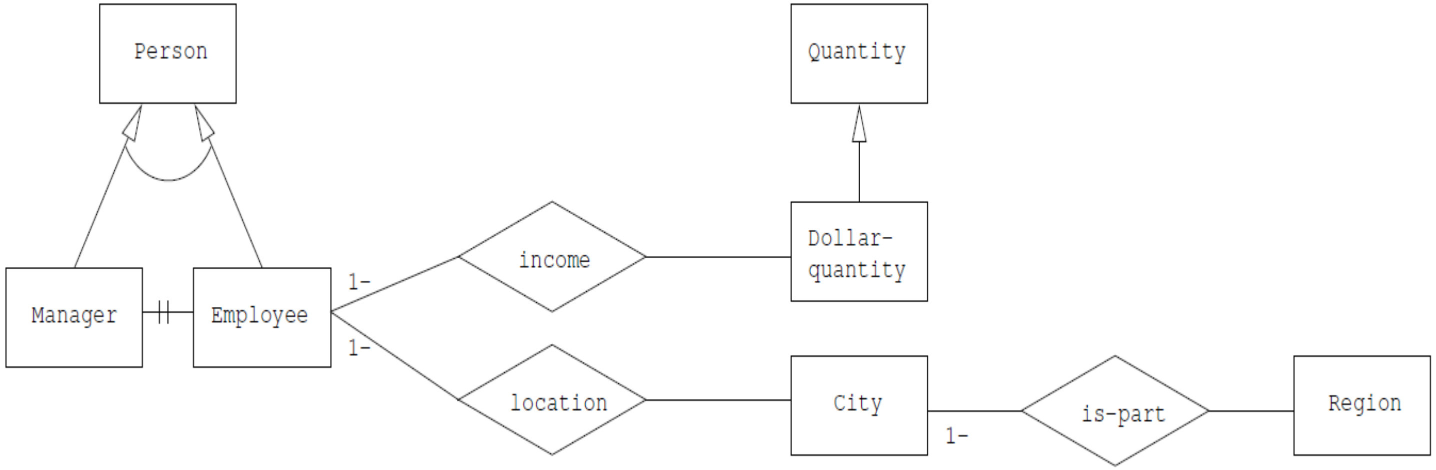
Observation. DBs use a table notation where, roughly speaking, rows describe **entities** plus their **properties** and **relations**, while columns name **etypes**, **entity properties** and **entity relations**. All types of percepts are part of the alphabet. The formation rules are those which allow to build tables.

Example – Entity graph alphabet and formation rules



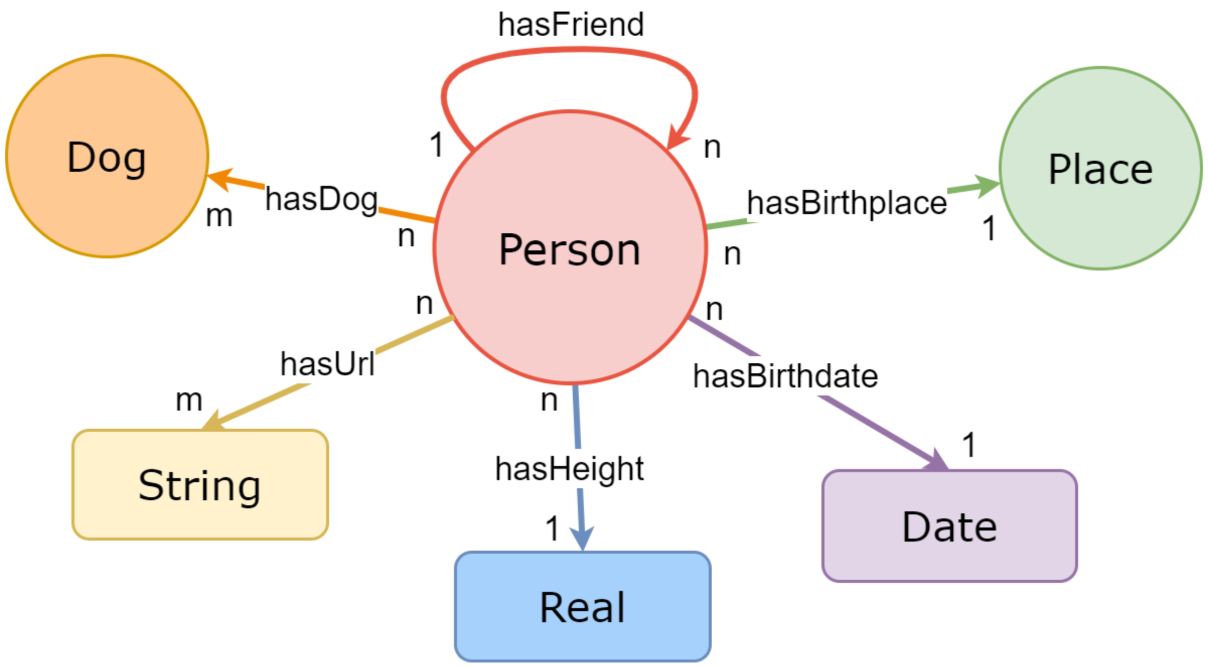
Observation. With respect to DB language: (1) same percepts and expressiveness, (2) the formation rules of a graph, more flexible that DB formation rules (3) closer than DBs to the analogical representation, (4) more complex to implement. (5) syntax and semantics are formalized (see later).

Example –ER model alphabet and formation rules



Observation. ER models use a graph notation with boxes of various shapes where the shape carries information about **etypes** and links carry information about **entity properties** and **relations**. Entities are not part of the percepts which can be mentioned as part of the language. Formation rules are formalized.

Example – Etype graph alphabet and formation rules



Observation (Etype Graph). With respect to ER models: (1) same percepts and expressiveness, (2) graph formation rule, more flexible than ER/EER models, (3) can be implemented (4) arity notation non-standard and not suggested (5) syntax and semantics are formalized (see later) (6) etype graphs implemented as knowledge level data structures (historically called «**ontologies**»).

Index

- From analogical to linguistic representations
- Logical languages
- Words, alphabet
- Assertional languages
- **Assertions**
- Assertional theories
- Key notions

From facts to assertions

Definition (Assertion). An **Assertion** a is an **atomic sentence**, that is, a sentence which cannot be decomposed into simpler sentences, which unambiguously describes a single **fact**. For any assertion a we have

$$a \in L_a = \{a\}$$

Observation (From facts to assertions) The mapping from facts to assertions is in full control of the modeler. We have the following:

- For formation rules there is no specific recipe. The general idea is that they should generate a syntax which makes it as easy as possible for people to understand the underlying facts (see above);
- A special case, see above, is when there is no need of formation rules.

Types of assertions

Definition (Assertion). An **assertion** a has one of the following five forms

- Assertion starting fact: $u_i \in C_j$,
- Assertion starting fact: *Tuple of Units memberOf relation:* $< u_1, \ldots, u_n > \in R^n$,
- Assertion starting fact: *Class subsetOf Class:* $C_i \subseteq C_j$,
- Assertion starting fact: *Relation subsetOf relation:* $R_i^n \subseteq R_j^n$
- Assertion starting fact: *Relation subsetOf tuple of classes and viceversa:*
  - $R^n \subseteq C_1 \times \ldots \times C_n$
  - $C_1 \times \ldots \times C_n \subseteq R^n$

Different assertional languages differ in the choice of facts to be asserted. See examples below.

Example – Natural language assertions

Example (Natural language, example assertions in yellow). "The origins of Trento on the river-route to Bolzano and the low Alpine passes of Brenner and the Reschen Pass over the Alps **are disputed**." Some scholars maintain "it was a Rhaetian settlement:" the "Adige area was however influenced by neighbouring populations", **including** the (Adriatic) Veneti, the Etruscans and the Gauls (a Celtic population). According to other theories, "the **latter** instead founded the city during the 4th century BC".

Observation (types of assertions). All types of assertions are allowed.

Example – Pseudo - natural language assertions

Example 1 (Pseudo-natural language). A sequence of statements:

- There is a tree.
- There is a banana.
- The monkey is eating a banana.
- Monkeys eat bananas.
- Monkeys are animals.

Observation (types of assertions). All types of assertions are allowed.

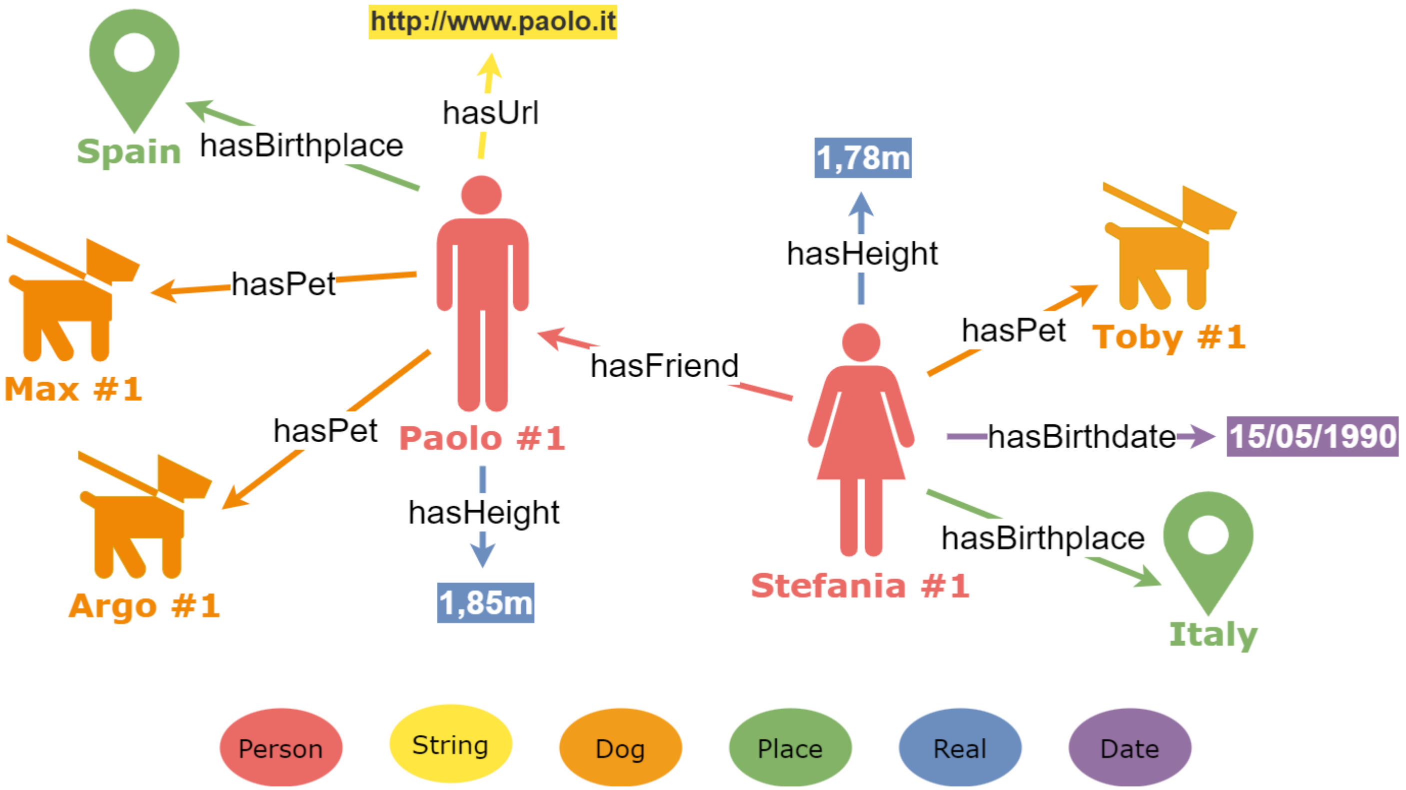
Example – Database assertions

Employee			
Name	**Role**	**Nationality**	**Supervises**
Fausto	Professor	Italian	Rui
Rui	Student	Chinese	Bisu
Bisu	Student	Indian	-
entity	etype	property	Relation

Observation.
DB assertions (in Logic) = {Professor(Fausto), Student(Rui), Student(Bisu), Nationality(Fausto, Italian), Nationality (Rui, Chinese), Nationality (Bisu, Indian), Supervises(Fausto, Rui), Supervises(Rui, Bisu)}

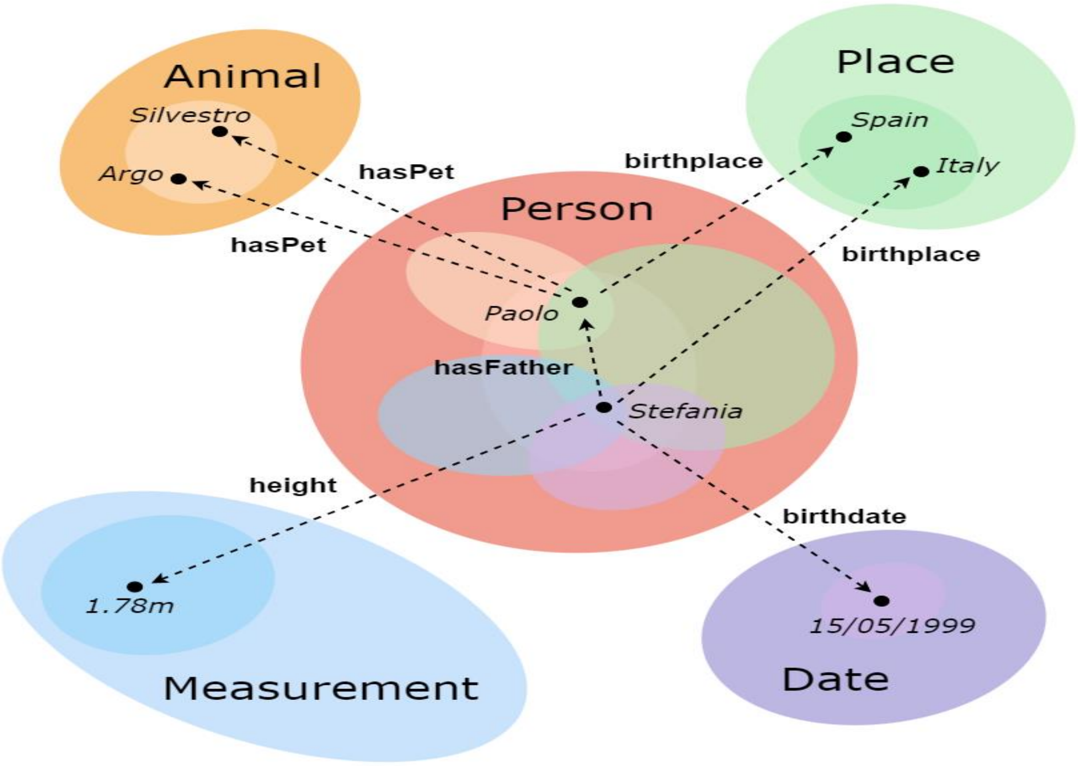
Observation (Types of assertions). Only assertions about entities. Etype assertions are **not** allowed in DBs.

Example – Entity graph assertions



Observation (Entity Graph). Same assertions as DBs.

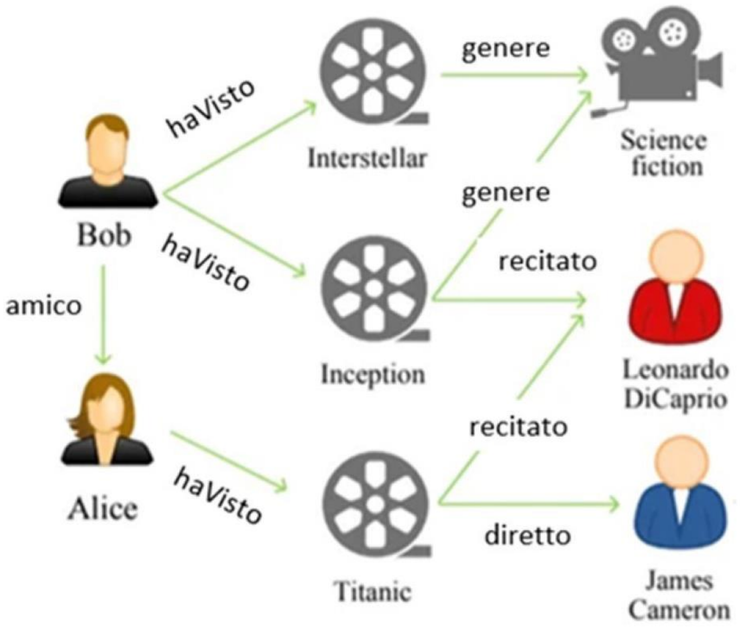
Example – Entity graph facts / assertions



Observation (Venn diagram of an entity Graph). A Venn diagram (analogic) representation of an entity Graph (non standard).

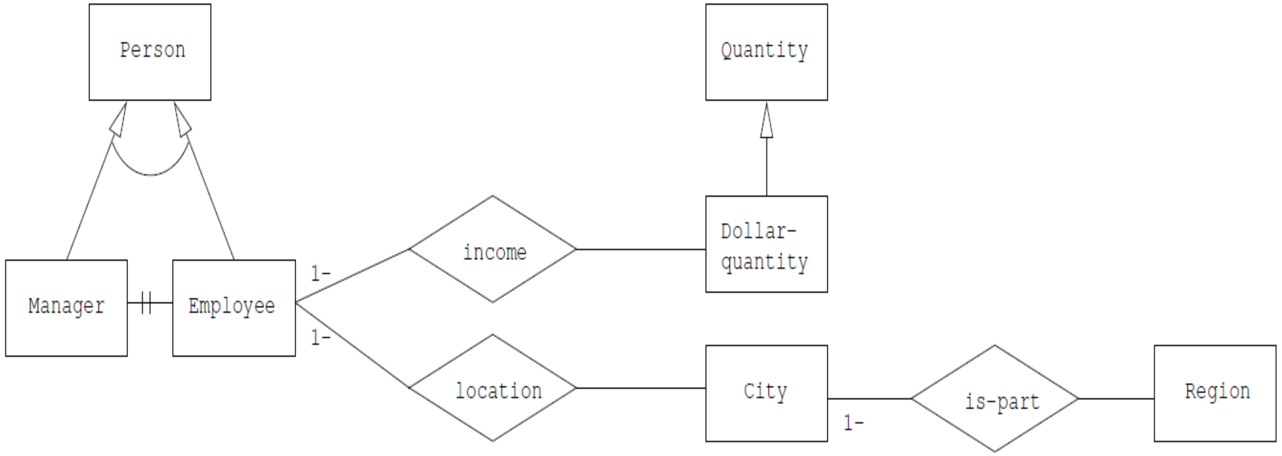
- Entities as nodes
- Etypes as sets of entities
- Relations as sets of tuples

Example – Knowledge graph assertions



Observation (Knowledge Graph KG). A KG is a graph whose language and semantics are not formalized and can be anything (no precise formation rules or definition of which percepts are allowed). In the graph above, all nodes are entities.

Example – ER assertions



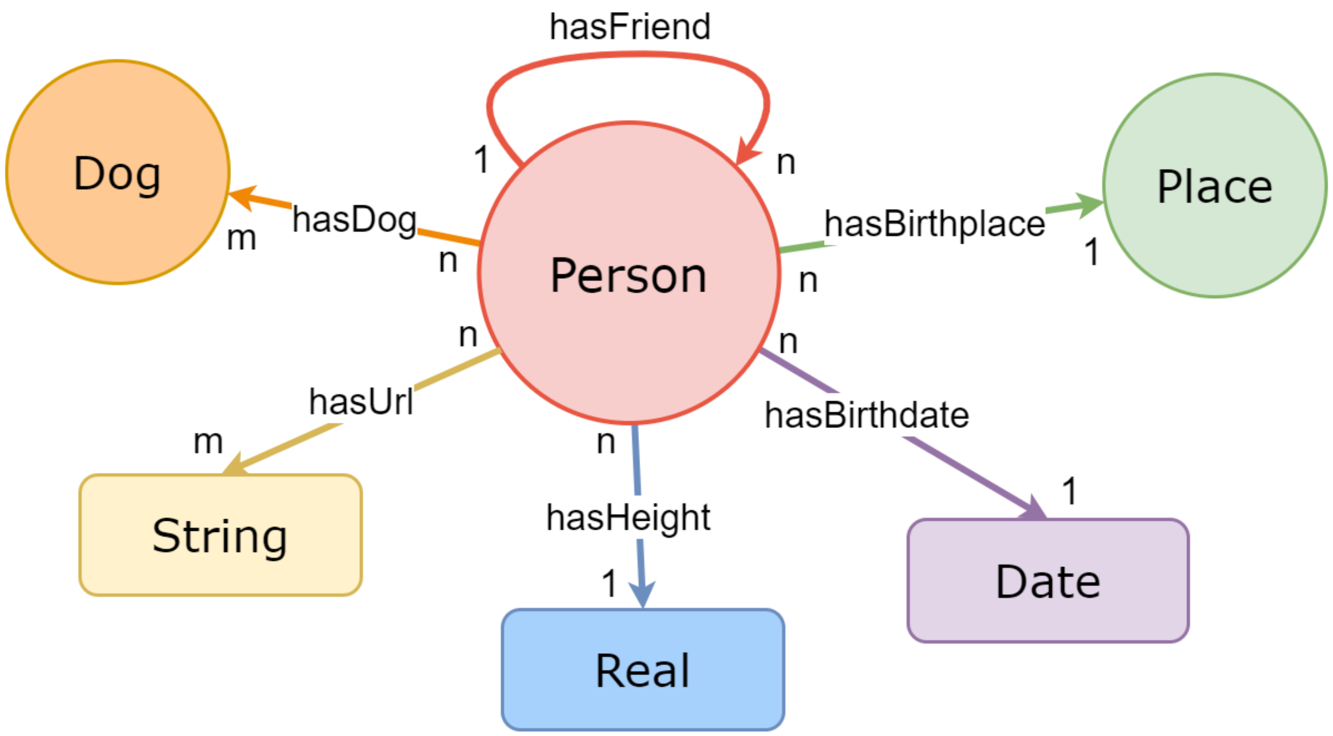
ER assertions relating to etypes (in Logic):

{Manager $\sqsubseteq$ Person, Employee $\sqsubseteq$ Person}

Observation (Intuitive semantics, examples). Professor is an etype, $\sqsubseteq$ semantically means subset, Fausto is a entity, Professor(Fausto) means that the etypes of Fausto is professor.

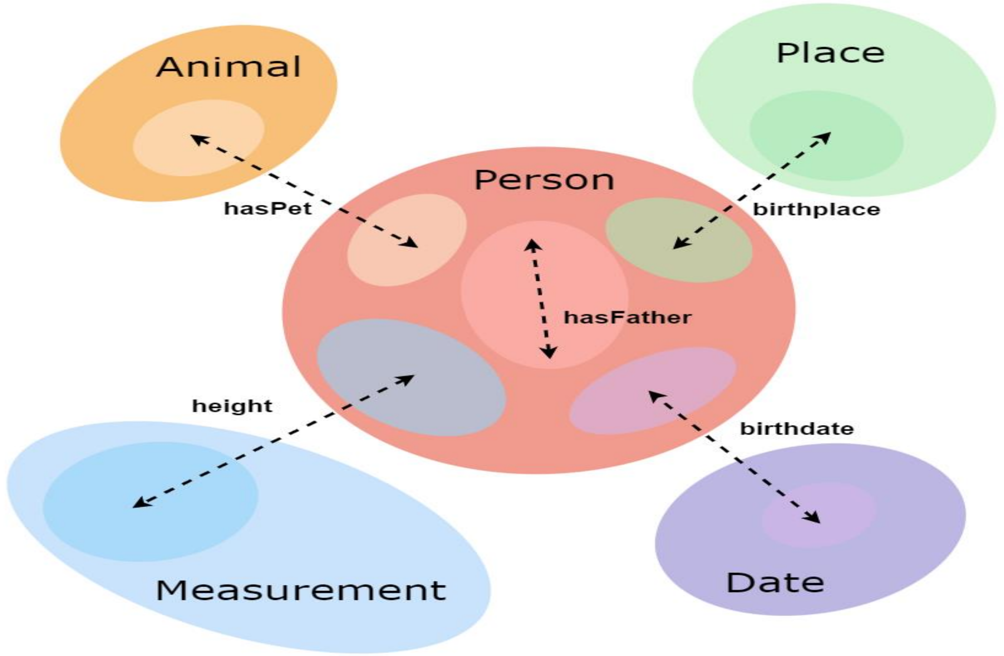
Observation (Types of assertions). Only assertions about etypes**.** Entity assertions are **not** allowed in ER models.

Example – Etype Graph assertions



Observation (Etype Graph). Same assertions as ER models.

Example – Etype Graph facts /assertions



Observation (Venn diagram of an entity Graph). A Venn diagram (analogic) representation of an entity Graph.

- No Entities
- Etypes as «zoomed» entities
- Relations as tuples of etypes

Observation (Etype Graph). A Venn diagram representation of an etype Graph. Not very immediate. Compare with the Venn diagram representation of the entity graph

Example – Concept assertions

Observation (Concept Graph CG). A snapshot of the **WordNet** output. Wordnet a **digitalized** lexicon which encodes how the meaning of words (concepts) is connected, so-called **Lexical-Semantics. Built to be used by machines»**

Example (linguistic assertions). Some examples (IsA means «subset», sameAs means «the same set»):

- Koto IsA stringed instrument
- Stringed instrument IsA musical instrument
- Musical instrument sameAs instrument
- Instrument IsA device
- ...
- Unit IsA Object
- Object sameAs physical object
- Physical object IsA Physical entity
- Physical entity IsA Entity
- ***Entity sameAs percept /* In this course */***

- S: (n) **koto** (Japanese stringed instrument that resembles a zither; has a rectangular wooden sounding board and usually 13 silk strings that are plucked with the fingers)
  - *direct hypernym* / ***inherited hypernym*** / *sister term*
    - S: (n) stringed instrument (a musical instrument in which taut strings provide the source of sound)
      - S: (n) musical instrument, instrument (any of various devices or contrivances that can be used to produce musical tones or sounds)
        - S: (n) device (an instrumentality invented for a particular purpose) *"the device is small enough to wear on your wrist"; "a device intended to conserve water"*
          - S: (n) instrumentality, instrumentation (an artifact (or system of artifacts) that is instrumental in accomplishing some end)
            - S: (n) artifact, artefact (a man-made object taken as a whole)
              - S: (n) whole, unit (an assemblage of parts that is regarded as a single entity) *"how big is that part compared to the whole?"; "the team is a unit"*
                - S: (n) object, physical object (a tangible and visible entity; an entity that can cast a shadow) *"it was full of rackets, balls and other objects"*
                  - S: (n) physical entity (an entity that has physical existence)
                    - S: (n) entity (that which is perceived or known or inferred to have its own distinct existence (living or nonliving))

Observation (Types of assertions). Only assertions about the which concept denotes a set of entities which is a subset of another one.

From Domains to Assertional Languages (continued)

Observation (From Domains to Assertional languages) When describing the intended model (described using model theory) the obvious way to define a language is as follows:

- For any percept p ∈ D= {p}, which is **of relevance** to the current modeling task, define a distinct word w ∈ A_a = {w};
- For any fact f ∈ D = {f}, which is **of relevance** to the current domain, construct a distinct assertion a ∈ L_a = {a}, where the set formation rules FR = {fr} allows to construct assertions following a process similar to that used in constructing facts from percepts.

From Domains to Assertional Languages (continued)

Observation (Domain) As from before, a Domain D is the set of all and only the **facts** f that can be composed from percepts, that is:

$$D = \{f\}$$

Observation (Assertional Language). Similarly to domains, an assertional language L_a is the set of all and only the sentences a which can be composed by applying the formation rules fr ∈ FR to the words in A_a, that is:

$$L_a = \{a\}$$

From Domains to Assertional Languages (continued)

Observation (Domain) As from before, a Domain D can be seen in two ways, as follows:

$$D = \langle U, \{C\}, \{R\} \rangle$$

$$D = \{f\}$$

The first is the **intensional** view, $\mathbf{D^i}$ of D the second is the **extensional** view, $\mathbf{D^e}$.

Observation (Assertional Language). Similarly to domains, an assertional language L_A can be seen in two ways, as follows:

$$L_a = \langle A_{\boldsymbol{a}}, FR \rangle$$

$$L_a = \{a\}$$

The first is the **intensional** view $\mathbf{L_a^i}$, of L_a, the second is the **extensional** view $\mathbf{L_a^e}$.

Index

- From analogical to linguistic representations
- Logical languages
- Words, alphabet
- Assertional languages
- Assertions
- **Assertional theories**
- Key notions

Assertional theories

Observation (Assertional theories). Assertions are linguistic descriptions of facts. **Assertional theories** are linguistic descriptions of models

Definition (Assertional theory) An **assertional theory** T_a is a set of assertions

$$\mathbf{T}_a = \{a\}$$

Observation (Assertional theory, assertional language). Given an assertional language L_a, an assertional theory T_a is a subset of L_a.

$$T_a = \{a\} \subseteq L_a$$

Example (Assertional theories). All the three examples above of sets of assertions are assertional theories.

Assertional theories - inconsistency

Intuition (Mutually inconsistent assertions). Two assertions a_1, a_2 are (**mutually**) **inconsistent** if they describe two facts which are mutually inconsistent.

Terminology (Inconsistency, consistency). If $a_1 \in T_a$, and if a_1, a_2 are two inconsistent assertions then we say that a_2 is **inconsistent with** T_a and that $\{a_2\} \cup T_a$ is **inconsistent**, and that $\{a_2\} \cup T_a$ **does not have a model**. Two assertional theories are (mutually) **inconsistent** if they contain two (mutually) inconsistent assertions. **Consistency** means absence of inconsistency.

Example – a DB generated Theory

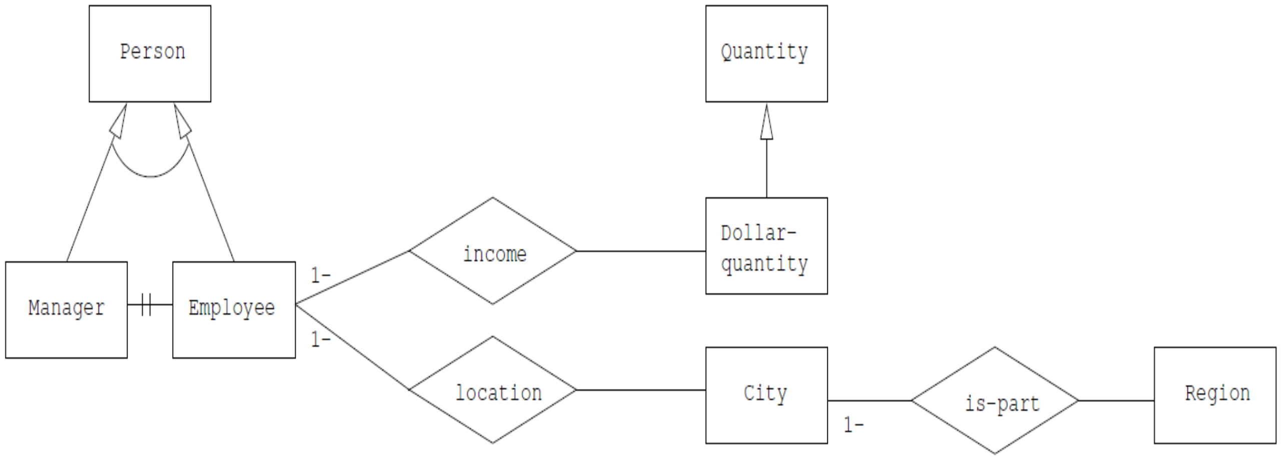
Employee

Name	Role	Nationality	Supervises
Fausto	Professor	Italian	Rui
Rui	Student	Chinese	Bisu
Bisu	Student	Indian	-
entity	etype	property	Relation

$\mathbf{T}_{a}$ =

{Professor(Fausto), Student(Rui), Student(Bisu),
Nationality(Fausto, Italian), Nationality (Rui, Chinese),
Nationality (Bisu, Indian), Supervises(Fausto, Rui)}

Example – An ER model generate theory



T_a =

{Manager ⊑ Person, …}

Index

- From analogical to linguistic representations
- Logical languages
- Words, alphabet
- Assertional languages
- Assertions
- Assertional theories
- **Key notions**

Set theory – Terminology

Terms naming components of linguistic representations

- *Entity*
- *Etype*
- *(Entity) Property*
- *Property (type)*
- *(Entity) Relation*
- *Relation (type)*
- *Word / (concept)*
- *Language*
- *Assertion*
- *Theory / (linguistic representation)*

<u>Corresponds to</u>

Set-theoretic terms naming components of analogic representations

- *Unity, element*
- *Class / set*
- *Tuple*
- *Relation, that is set of tuples*
- *Tuple*
- *Relation, that is set of tuples*
- *Percept*
- *Domain (of interpretation)*
- *Fact*
- *Model / (analogic representation)*

Each terms on the left (e.g., entity) is the name of the element which plays the same role as the element whose name is in the same line, on the right (e.g., element).

Notation (terms used). In informal writing, whenever no confusion arises we will use terms on the right in place of terms on the left, to facilitate the interpretation. Moreover we will drop the terms in parenthesis.

Key notions

- Words, concepts
- Alphabet
- Assertions
- Languages, logical languages, assertional languages
- Assertional theories
- Inconsistency, assertions, theories
- Examples of world models:
  - DBs,
  - ER/ EER models,
  - KGs,
  - Entity graphs, etype graphs,
  - Natural Language, pseudo-natural language,
  - Concept Graphs (CGs), i.e., Digital lexicons

Theories

(of the world)

(HP2T)